COMP3153/9153
Homework 1

Temporal Logic, CTL Model Checking, Büchi Automata

Due: Fri 20th March 2020, 10am
Submission guidelines are given at the end of this document.

Exercise 1 (LTL – Part I) (20 Marks)

Assume the set of atomic propositions is $\{a, b, c\}$.

Question 1 For each of the following set of paths $S_i, 1 \leqslant i \leqslant 5$, give an LTL formula, φ_i, that best defines S_i (and justify your answer):

1. S_1: paths that contain at least two a;
2. S_2: paths that do not contain a state satisfying b and $\neg a$;
3. S_3: paths that contain at least one c which is not immediately followed neither by a nor b;
4. S_4: paths that eventually do not contain a a.
5. S_5: paths that satisfy: if containing a finite number of a then they do not contain an infinite number of c.

Note: you can use the operators **F** and **G**, as well as all other LTL-operations.

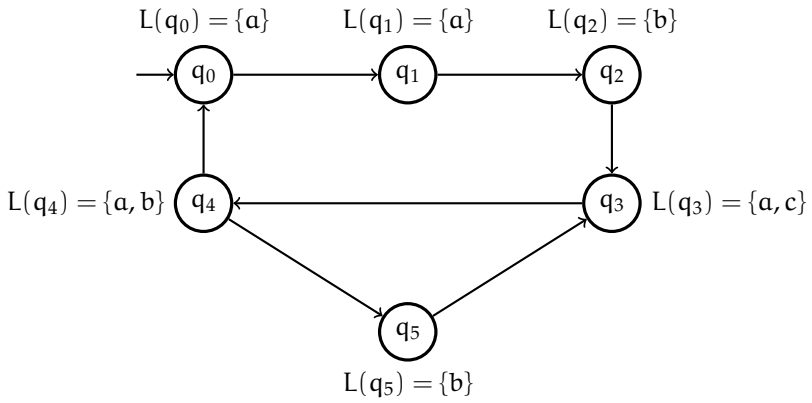


Figure 1: Automaton A

Question 2 Given the labeled automaton A of Fig. 1, with labeling-function L.

For each of the following LTL formulas, is there at least one run/trace in A that satisfies the formula? If you answer "Yes" give a witness, and if you answer "No" justify your answer.

1. $\mathbf{X}\,\mathbf{X}\,b$;
2. $\mathbf{F}\,c$;
3. $\mathbf{G}\,\mathbf{F}\,\mathbf{G}\,(b \vee a)$;
4. $\mathbf{F}\,\mathbf{G}\,((\mathbf{X}\,\neg b) \vee b)$;
5. $\mathbf{G}\,(a\ \mathbf{UNTIL}\ (\neg a \wedge b))$.

Question 3 Among the LTL formulas (1–5) of Question 2, which are the ones satisfied by automaton A? (Explain your answer).

Exercise 2 (CTL – Part I) (10 Marks)

A system is composed of two tanks, A and B, and a pump Pump. The states of the system are labelled by atomic propositions from the set

{pumpOn, pumpOff, tankAEmpty, TankAFull, TankBEmpty, tankBFull}

Question 4 Consider the following informal requirements and CTL formulas that should formally define the corresponding requirements:

1. *"It is always the case that if the pump is off, it will be on again in the future."*
   CTL formula: $\mathbf{AG}\,(\text{pumpOff} \Rightarrow \mathbf{AF}\,\text{pumpOn})$
2. *"The pump is always off if tank A is empty or tank B is full".*
   CTL formula: $\mathbf{AG}\,\mathbf{AF}\,(\text{pumpOff} \Rightarrow (\text{tankAEmpty} \vee \text{tankBFull}))$
3. *"It is always possible to reach a state where tank B is full".*
   CTL formula: $\mathbf{AG}\,\mathbf{EF}\,\text{tankBFull}$

One of the previous CTL formulas does not match the informal requirement. Which one? (Explain your answer).

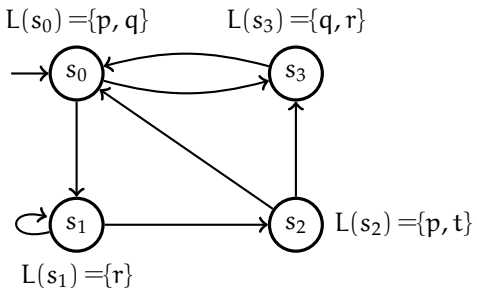


Figure 2: Automaton B

Question 5 What would be the correct CTL formula corresponding to the informal requirement?

Exercise 3 (CTL — Part II) (20 Marks)

Question 6 Consider the automaton B of Figure 2. Check whether, $B, s_0 \models \phi$ and $B, s_2 \models \phi$ for the following CTL formulas ϕ:

1. **AF** q
2. **AG**(**EF** $(p \vee r)$)
3. **EX**(**EX** r)
4. **AG**(**AF** q)

Question 7 Which of the following pairs of CTL formulas are equivalent. For those that are not equivalent, find a model of one of the pair which is not a model of the other.

1. **EF** ϕ and **EG** ϕ
2. (**EF** ϕ) $\vee$ (**EF** ψ) and **EF** $(\phi \vee \psi)$
3. (**AF** ϕ) $\vee$ (**AF** ψ) and **AF** $(\phi \vee \psi)$
4. **AF** $\neg\phi$ and $\neg$**EG** ϕ
5. true and (**AG** ϕ) $\Rightarrow$ (**EG** ϕ)
6. true and (**EG** ϕ) $\Rightarrow$ (**AG** ϕ)

Exercise 4 (CTL Model Checking) (30 Marks)

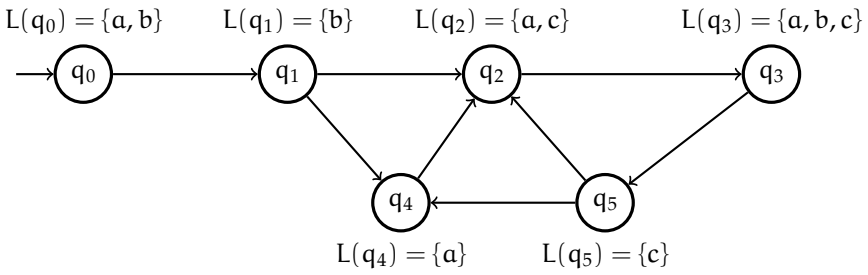


Figure 3: Automaton C

Question 8 Let φ be the following CTL formula:

$$\mathbf{AG}\left(b \Rightarrow \mathbf{AX}\neg\,\mathbf{E}\big((a \vee c)\ \mathbf{UNTIL}\ b\big)\right).$$

1. Rewrite the formula φ into equivalent formula φ' without **AG**, **AX** and $\Rightarrow$. We consider that the *or* operator, $\vee$, is allowed in a CTL formula.
2. Give the parse tree of the rewritten formula φ'.
3. Manually run the CTL explicit-state marking algorithm from Week 2 on φ' i.e., compute the result of $\mathsf{Mark}(C, \varphi')$, to determine whether $C \models \varphi'$ (and explain your answer).

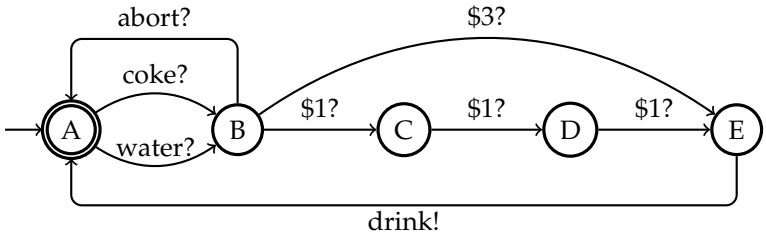
Exercise 5 (Büchi Automata) (20 Marks)

Figures 4(a) and 4(b) model a vending machine and a customer, respectively. The automata are *Büchi automata*.

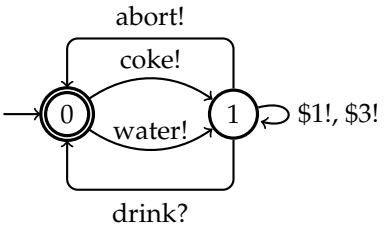
The two automata synchronise on matching send/receive actions: α!/α? results in a common action of the components named α.

Question 9 Briefly describe the behaviour of the automata in English.

Question 10 Build the synchronised product *Vending Machine2* × *Customer2*.



(a) Automaton *Vending Machine2*



(b) Automaton *Customer2*

Figure 4: Automata for Exercise 5

Submission Guidelines

- Due time: Fri 20th March 2020, 10am. **No late submission allowed**.
- Submit one PDF file (`hw1.pdf`) using the CSE `give` system by typing the command `give cs3153 hw1 hw1.pdf` on a CSE terminal. Alternatively use the online submission page.
- It is highly recommended that you use LaTeX to prepare your document. A guide is provided on the course website.
